THE ANNALS OF UNIVERSITY “DUNĂREA DE JOS” OF GALAȚI
FASCICLE VIII, 2002, ISSN 1221-4590
TRIBOLOGY

THE TOLERANCE FIELD EFFECT ON THE ANGULAR CONTACT BALL BEARINGS SYSTEMS’ RATING LIFE

Rezmires Daniel, Racocea Cezar

Technical University "Gh. Asachi” of Iasi, drezmir@yahoo.com

ABSTRACT

To assure both good dynamic load capacity and higher shaft stiffness, two rolling bearings are usually mounted in pair. The load distribution on the contacts of the two rolling bearings depends on individual stiffness of the each bearing, on the length separator between the bearings, and on the chosen tolerance values. In this work we present a model in five degrees of freedom, which could serve to find the load distribution in bearing arrangements, considering the intermediate elements as rigid bodies. The assembly’ stiffness was determined considering the individual stiffness of each element, and the rating life was expressed as a function of tolerance values of the intermediate elements. This was realised by solving a non-linear system of equations, including the centrifugal effects and some of the friction forces.

KEYWORDS: Angular – contact ball bearing, Quasi-dynamic equilibrium, Rating life.

1.Analytical Approach

For any rolling bearing pair, (r,j), the distance pieces L1, L2 are considered to have the same initial length and the shaft is considered as rigid.

The following co-ordinate systems were considered:

- an inertial system $OXYZ$ with its origin on the middle length of the inner ring's curvature centres;
- a rolling element frame $OX_1Y_1Z_1$ for each (r,j) ball.

To limit the complexity of the analysis, the following assumptions were admitted:

- the bearing was mounted on an elastic shaft and in a rigid housing;
- the surfaces in contact have ideal shapes;
- the pair bearing system were considered to be rigid except the local contact zones.

Related to the ball co-ordinate system $(OX_1Y_1Z_1)_{r,j}$ two degrees of freedom, represented by the translations ux and uz, have been considered for each ball.

The external load vector $\{F\}$, applied to the entire arrangement, contains 5 components that are further divided to each bearing of the arrangement:

$$\{F\} = \{F_x,F_y,F_z, M_y, M_z\} \quad (1a)$$

$$\{F\}_r = \{F_{xr},F_{yr},F_{zr}, M_{yr}, M_{zr}\} \quad (1b)$$

The static equilibrium provides easily the system of equations:

$$\begin{array}{ll} F_{x1}+F_{x2}=F_x & \\ F_{rz2}+F_{rz1}=F_z & F_{ry2}+F_{ry1}=F_{ry} \\ F_z\cdot(L+B)=F_{rz1}.B & F_y\cdot(L+B)=F_{ry1}.B \\ M_{y1}+M_{y2}=M_y & M_{z1}+M_{z2}=M_z \end{array} \quad (2)$$

The displacement vector $\{\delta\}_r$ of the “r” inner ring has also five components:

$$\{\delta\}_r = \{\delta_x, \delta_{yr}, \delta_{zr}, \gamma_y, \gamma_z\} \quad (3)$$

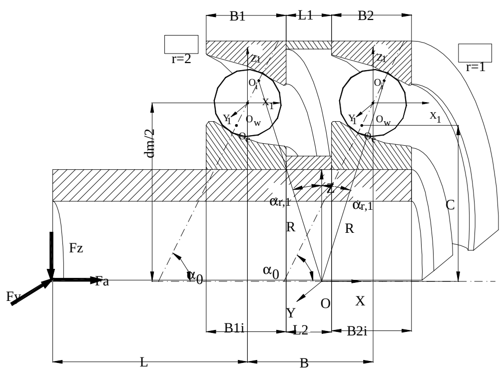

Fig.1. General view

2. Static Equilibrium of the (*r,j*) ball element

To solve the equilibrium system (3), is necessary to find the components of $\{\delta\}_r$ vector which are functions of distances O_eO_w, and O_iO_w respectively. The following notations were introduced:

$$l_{oe}= O_eO_w= Ro-D_w/2-Sd/4; \quad (4a)$$

$$l_{oi}= O_iO_w =Ri-D_w/2-Sd/4; \quad (4b)$$

$$L_{ie}=l_{oi}+l_{oe} \quad (4c)$$

Considering identical bearings in the arrangement pair, the $D1$ and $D2$ values presented in Figures 2 and 3, are :

$$D1 = L1 + (B_{i,1} + B_{i,2})/2 \tag{5a}$$
$$D2 = L2 + (B_{o,1} + B_{o,2})/2 \tag{5b}$$
$$D1 = D2 \tag{5c}$$

Considering further the tolerances $TB_{i,o,r}$ corresponding to $B1$, $B2$ distances and the tolerances $TL_{1,2}$ corresponding to $L1$ and $L2$ distances, the values $D1$ and $D2$ become:

$$D_{2p} = L2 + TB_{o,1} + TL_2 + TB_{o,2} + (B_{o,1} + B_{o,2})/2 \tag{6}$$
$$D_{1p} = L1 + TB_{i,1} + TL_1 + TB_{i,1} + (B_{i,1} + B_{i,2})/2 \tag{7}$$

The initial contact angle α_0 and L_{ie} parameters depend also on D_{2p} and D_{1p} values, so that new values $\alpha_{0,r}$ and $L_{ie}(r)$ have to be considered. Also, $\alpha_{1,r}$ and R parameters are different versus the initial values.
The supplementary axial clearance introduced by the effective values for D_{2p} and D_{1p} parameters is:

$$ja = D_{2p} - D_{1p} \tag{8}$$

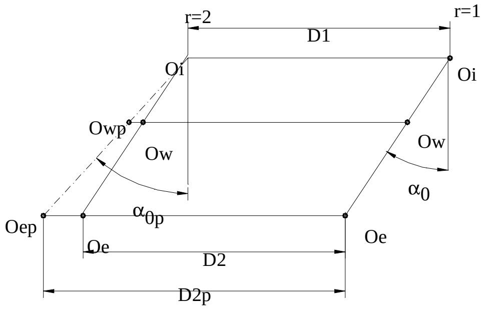

Fig. 2. The α_0 and L_{ie} when $L2+(B_{o,1}+B_{o,2})/2 > L1+(B_{i,1}+B_{i,2})/2$

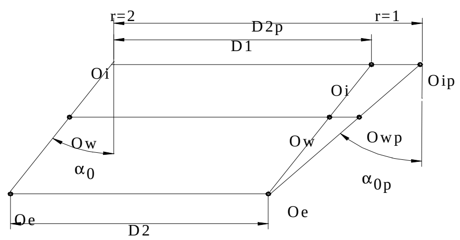

Figure 3. The α_0 and L_{ie} when $L2+(B_{o,1}+B_{o,2})/2 < L1+(B_{i,1}+B_{i,2})/2$

Assuming "ja" as decision criteria results:
if $ja \geq 0$

- for $r=2$
- $\alpha_{0,2} = arctan((L_{ie}.sin(\alpha_0)+ja)/(L_{ie}.cos(\alpha_0))$;
- $sd1(2)=[(L_{ie}.cos(\alpha_{0,2}))^2+(L_{ie}.sin(\alpha_{0,2})+ja)^2]^{0.5}-L_{ie}$;
- $L_{ie}(2) = loi+loe+sd1(2)$;
  - for $r=1$
  - $\alpha_{0,1}=\alpha_0$; $sd1(1):=0$; $L_{ie}(1):=L_{ie}$

and

- $\alpha_{r,1}=arctan\{[D_{2p}]/[2.[dm/2+loi(r).cos(\alpha_{0,r})]]\}$
- $R=[D_{2p}]/[2.sin(\alpha_{r,1})]$;

if $ja<0$

- for $r=1$
- $\alpha_{0,1} = arctan((L_{ie}.sin(\alpha_0)+ja)/(L_{ie}.cos(\alpha_0))$;
- $sd1(1)=[(L_{ie}.cos(\alpha_{0,1}))^2+(L_{ie}.sin(\alpha_{0,1})+ja)^2)^{0.5}-L_{ie}$;
- $L_{ie}(1)=loi+loe+sd1(1)$;
  - for $r=2$
  - $\alpha_{0,2}=\alpha_0$; $sd1(2):=0$; $L_{ie}(2):=L_{ie}$

and

- $\alpha_{r,1}=arctan\{[D_{2p}]/[2.[dm/2+loi(r).cos(\alpha_{0,r})]]\}$
- $R=[D_{2p}]/[2.sin(\alpha_{r,1})]$;

If the inner ring is misaligned around OY and Oz axes with γ_y and γ_z angles respectively, then the initial angle $\alpha_{r,1}$ become a function of $\alpha_1(r,j)$:

$$\alpha_1(r,j)=\alpha_{r,1}+sgn(r).\gamma_y.cos(\psi(r,j))+sgn(r).\gamma_z.sin(\psi(r,j)) \tag{9}$$

where:

- $\psi(r,j)$ defines the angular position of the ball element in the inertial system;
- $sgn(r)$ defines "r" row:
- $$sgn(r) = \begin{cases} 1, & r=1 \\ -1, & r=2 \end{cases} \tag{10}$$

Because in the static load case the inner and outer contact angles are equal for any individual ball element, but different for every ball, the total deformation that acts on the (r,j) ball can be written as:

$$\delta(r,j)=\sqrt{x(r,j)^2+z(r,j)^2}-l_{oi}-l_{oe} \tag{11}$$

where:

$$z(r,j)=L_{ie}(r).cos(\alpha_{0,r})+\delta_{zr}.cos(\psi(r,j))+$$
$$+\delta_{yr}.sin(\psi(r,j))+R.[cos(\alpha_1(r,j))-cos(\alpha_{r,1})]$$
$$x(r,j)=L_{ie}(r).sin(\alpha_{0,r})+\delta_x+R[sin(\alpha_{r,1})-$$
$$-sin(\alpha_1(r,j))]$$

The contact angle for the (r,j) roller element is:

$$\alpha_s(r,j)=\alpha_i(r,j)=\alpha_e(r,j)=arctan\left(\frac{x(r,j)}{z(r,j)}\right) \tag{12}$$

The normal load and contact angle are given by:

$$Q(r,j) = K_{ech}.\delta(r,j)^n \tag{13a}$$
$$\alpha_i(r,j) = \alpha_e(r,j) \tag{13b}$$

The $\{\delta\}_r$ displacement vector results by solving the equilibrium equation system for the inner ring. Using the previous relations, the equilibrium of forces and moments are:

$$F_z=\sum_r\sum_j Q(r,j).cos(\alpha_i(r,j))cos(\psi(r,j)) = \sum_r\sum_j F_{zr}(r,j) \tag{15a}$$

$$F_y=\sum_r\sum_j Q(r,j).cos(\alpha_i(r,j))sin(\psi(r,j)) = \sum_r\sum_j F_{yr}(r,j) \tag{15b}$$

$$F_x = \sum_r \sum_j Q(r,j) sin(\alpha_i(r,j)) = = \sum_r \sum_j F_x(r,j) \tag{15c}$$

$$M_y = \sum_r \sum_j F_x(r,j).b_y(r,j) + + \sum_r \sum_j F_{zr}(r,j).b_x(r,j). \tag{15d}$$

$$M_z = \sum_r \sum_j F_x(r,j).b_z(r,j) + + \sum_r \sum_j F_{yr}(r,j).b_x(r,j). \tag{15e}$$

where:

- $Q(r,j)$ represents the load acting on the (r,j) ball;
- $F_{zr}(r,j)$, $F_{yr}(r,j)$ represent the radial forces which act in "r,j" ball;
- $b_{x,y,z}(r,j)$, represents the distance from the point of inner raceway - ball contact to the center of the inertial system.

$$b_x(r,j) = \frac{B}{2} + \left(\delta_i(r,j) + l_{oi} - \frac{D_w}{2}\right) sin(\alpha_s(r,j))$$

$$b_y(r,j) = \left[C + \left(\delta_i(r,j) + l_{oi} - \frac{D_w}{2}\right) cos(\alpha_s(r,j))\right]. .sin(\psi(r,j))$$

$$b_z(r,j) = \left[C + \left(\delta_i(r,j) + l_{oi} - \frac{D_w}{2}\right) cos(\alpha_s(r,j))\right]. .cos(\psi(r,j))$$

$$\delta_i(r,j) = \delta(r,j).(K_{ech}/K_i)^{1/n} \tag{16}$$

The $\{\delta\}_r$ components represent the solution of the Eq. (15a-15e) and it was found by an Newton-Raphson algorithm. To solve the equilibrium system (15) is necessary to write the Jacobian matrix for the two bearings. The rigidity matrix M_r is:

$$M_r = \begin{bmatrix} \frac{\partial Fxr}{\partial \delta x} & \frac{\partial Fxr}{\partial \delta y} & \frac{\partial Fxr}{\partial \delta z} & \frac{\partial Fxr}{\partial \gamma y} & \frac{\partial Fxr}{\partial \gamma z} \\ \frac{\partial Fry}{\partial \delta x} & \frac{\partial Fry}{\partial \delta y} & \frac{\partial Fry}{\partial \delta z} & \frac{\partial Fry}{\partial \gamma y} & \frac{\partial Fry}{\partial \gamma z} \\ \frac{\partial Frz}{\partial \delta x} & \frac{\partial Frz}{\partial \delta y} & \frac{\partial Frz}{\partial \delta z} & \frac{\partial Frz}{\partial \gamma y} & \frac{\partial Frz}{\partial \gamma z} \\ \frac{\partial My}{\partial \delta x} & \frac{\partial My}{\partial \delta y} & \frac{\partial My}{\partial \delta z} & \frac{\partial My}{\partial \gamma y} & \frac{\partial My}{\partial \gamma z} \\ \frac{\partial Mz}{\partial \delta x} & \frac{\partial Mz}{\partial \delta y} & \frac{\partial Mz}{\partial \delta z} & \frac{\partial Mz}{\partial \gamma y} & \frac{\partial Mz}{\partial \gamma z} \end{bmatrix}_r \tag{17}$$

where:

$$\frac{\partial Fxr}{\partial\{\delta\}_r} = \sum_j \frac{\partial[K_{ech}.\delta(r,j)^n.sin(\alpha_i(r,j))]}{\partial\{\delta\}_r} \tag{18a}$$

$$\frac{\partial Fry}{\partial\{\delta\}_r} = \sum_j \frac{\partial[K_{ech}.\delta(r,j)^n.cos(\alpha_i(r,j)).sin(\psi(r,j))]}{\partial\{\delta\}_r} \tag{18b}$$

$$\frac{\partial Frz}{\partial\{\delta\}_r} = \sum_j \frac{\partial[K_{ech}.\delta(r,j)^n.cos(\alpha_i(r,j)).cos(\psi(r,j))]}{\partial\{\delta\}_r} \tag{18c}$$

$$\frac{\partial M_y}{\{\delta\}_r} = \frac{\partial \sum_j F_x(r,j).b_y(r,j) + \partial \sum_j F_z(r,j).b_x(r,j)}{\partial\{\delta\}_r}. \tag{18d}$$

$$\frac{\partial M_z}{\{\delta\}_r} = \frac{\partial \sum_j F_x(r,j).b_z(r,j) + \partial \sum_j F_y(r,j).b_x(r,j)}{\partial\{\delta\}_r}. \tag{18e}$$

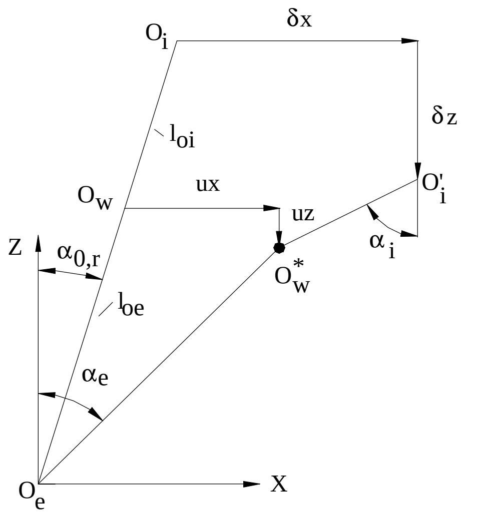

Fig. 4. The center of mass displacement (ux, uz) for the (r,j) ball.

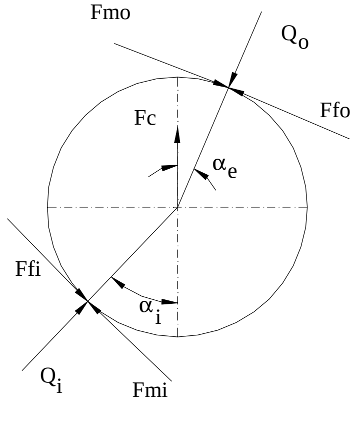

Fig. 5 The forces on (r,j) ball

3. Quasi-dynamic effects

Due to centrifugal force, both the load and the contact angle are modified versus the static values.

Considering the existence of the centrifugal force the final position for the mass centre of the *(r,j)* ball is presented in Fig. 4 as function of “*ux*” and “*uz*“ parameters.

The new position of O_{we} point is found also with the Newton Raphson algorithm applied this time to all balls. The loads that act on the *(r,j)* ball are presented in Fig.5.

Considering the guiding ball assumption [1], the equilibrium equations for the *(j)* ball are:

$$EFCA(r,j)=Q_i(r,j).sin(\alpha_i(r,j))-Q_o(r,j).sin(\alpha_e(r,j))-[Fmi.cos(\alpha_i(r,j))-Fmo.cos(\alpha_e(r,j))]=0 \quad (19a)$$

$$EFCR(r,j)=Q_i(r,j).cos(\alpha_i(r,j))-Q_o(r,j).cos(\alpha_e(r,j))+[Fmi.sin(\alpha_i(r,j))-Fmo.sin(\alpha_e(r,j))]+Fc=0 \quad (19b)$$

where:

$Fmi=2.(1-\lambda).Mg/Dw;$

$Fmo=2.\lambda.Mg/Dw;$

$Ffo=\mu.Qo;$

$Ffi=\mu.Qi$

$Mg=Dw^2.m.10^{-7}.\omega_c\,.\omega_w\,.\,sin(\beta)$

β - atitude angle, [rad]

$$\beta = arctan\left(\frac{sin(\alpha_e(r,j))}{cos(\alpha_e(r,j))+\gamma}\right)$$ and $\lambda=1$, for outer race guiding

$$\beta = arctan\left(\frac{sin(\alpha_i(r,j))}{cos(\alpha_i(r,j))-\gamma}\right)$$ and $\lambda=0$, for inner race guiding

The following must be considered:

- *Ffo* and *Ffi* act like blocking forces;
- If *Fmi>Ffi* then *Fmi=Fmi-Ffi* else *Fmi=0;*
- If *Fmo>Ffo* then *Fmo=Fmo-Ffo* else *Fmo=0*;

In these conditions the rigidity matrix for *(r,j)* element is:

$$MFC(r,j)=\begin{bmatrix}\dfrac{\partial EFCA(r,j)}{\partial ux} & \dfrac{\partial EFCA(r,j)}{\partial uz}\\ \dfrac{\partial EFCR(r,j)}{\partial ux} & \dfrac{\partial EFCA(r,j)}{\partial uz}\end{bmatrix} \quad (20)$$

The following notations were introduced to simplify the *MFC(r,j)* components:

dto=dto(r,j) - contact deformation for static load case at outer contact level;

dti=dti(r,j) - contact deformation for static load case at inner contact level;

$\alpha s=\alpha s(r,j)$ - contact angle in the static case;

$$ZO=(l_{oe}+dto).cos(\alpha s)+uz; \quad (21a)$$

$$XO=(l_{oe}+dto).sin(\alpha s)+ux; \quad (21b)$$

$$ZI=(l_{oi}+dti).cos(\alpha s)-uz; \quad (22a)$$

$$XI=(l_{oi}+dti).sin(\alpha s)-ux; \quad (22b)$$

$rri=(ZI^2+XI^2)^{0.5}-l_{oi};$ $\quad rro=(ZO^2+XO^2)^{0.5}-l_{oe}$

$Tqi=Ki.rri^{1.5}$ $\quad Tqo=Ko.rro^{1.5}$

$Tdi=1+(XI/ZI)^2$ $\quad Tdo=1+(XO/ZO)^2$

From these notations results:

$$\frac{\partial EFCAi}{\partial ux}=Tqi.\left[\frac{-1.5.XI^2}{rri.(rri+loi).ZI.\sqrt{Tdi}}-\frac{1}{ZI.\sqrt{Tdi}}+\frac{XI^2}{ZI^3.Tdi^{1.5}}\right]$$

$$\frac{\partial EFCAo}{\partial ux}=Tqo.\left[\frac{1.5.XO^2}{rro.(rro+loe).ZO.\sqrt{Tdo}}+\frac{1}{Zo.\sqrt{Tdo}}-\frac{XO^2}{ZO^3.Tdo^{1.5}}\right]$$

$$\frac{\partial EFCA}{\partial ux}=\frac{\partial EFCAi}{\partial ux}-\frac{\partial EFCAo}{\partial ux} \quad (23)$$

$$\frac{\partial EFCAi}{\partial uz}=Tqi.\left[\frac{-1.5.XI.ZI}{rri.(rri+loi).ZI.\sqrt{Tdi}}-\frac{XI}{ZI^2.\sqrt{Tdi}}+\frac{XI^3}{ZI^4.Tdi^{1.5}}\right]$$

$$\frac{\partial EFCAo}{\partial uz}=Tqo.\left[\frac{1.5.XO.ZO}{rro.(rro+loe).ZO.\sqrt{Tdi}}+\frac{XO}{ZO^2.\sqrt{Tdo}}+\frac{XO^3}{ZO^4.Tdo^{1.5}}\right]$$

$$\frac{\partial EFCA}{\partial uz}=\frac{\partial EFCAi}{\partial uz}-\frac{\partial EFCAo}{\partial uz} \quad (24)$$

$$\frac{\partial EFCRi}{\partial ux}=Tqi.\left[\frac{-1.5.XI}{rri.(rri+loi).\sqrt{Tdi}}+\frac{XI}{ZI^2.Tdi^{1.5}}\right]$$

$$\frac{\partial EFCRo}{\partial ux}=Tqo.\left[\frac{1.5.XO}{rro.(rro+loe).\sqrt{Tdo}}-\frac{XO}{ZO^2.Tdo^{1.5}}\right]$$

$$\frac{\partial EFCR}{\partial ux}=\frac{\partial EFCRi}{\partial ux}-\frac{\partial EFCRo}{\partial ux} \quad (25)$$

$$\frac{\partial EFCRi}{\partial uz}=Tqi.\left[\frac{-1.5.ZI}{rri.(rri+loi).\sqrt{Tdi}}-\frac{XI^2}{ZI^3.Tdi^{1.5}}\right]$$

$$\frac{\partial EFCRo}{\partial uz}=Tqo.\left[\frac{1.5.ZO}{rro.(rro+loe).\sqrt{Tdo}}+\frac{XO^2}{ZO^3.Tdo^{1.5}}\right]$$

$$\frac{\partial EFCR}{\partial uz}=\frac{\partial EFCRi}{\partial uz}-\frac{\partial EFCRo}{\partial uz} \quad \text{……..(26)}$$

4. The rigidity matrix of the "*r*" bearing

The displacements *ux* and *uz* are obtained solving the eq. (20) to (26). The rigidity matrix (17) has the following elements:

$$\frac{\partial Fx}{\partial\{\delta\}_r}=\sum_j \frac{\partial[K_i.\delta_i(r,j,ux,uz)^n.sin(\alpha_i(r,j,ux,uz))]}{\partial\{\delta\}_r}$$

$$\frac{\partial Fry}{\partial\{\delta\}_r}=\sum_j \frac{\partial[K_i.\delta_i(r,j,ux,uz)^n.cos(\alpha_i(r,j,ux,uz)).sin(\psi(r,j))]}{\partial\{\delta\}_r}$$

$$\frac{\partial Frz}{\partial\{\delta\}_r}=\sum_j \frac{\partial[K_i.\delta_i(r,j,ux,uz)^n.cos(\alpha_i(r,j,ux,uz)).cos(\psi(r,j))]}{\partial\{\delta\}_r}.$$

$$\frac{\partial M_y}{\{\delta\}_r}=\frac{\partial\sum_j F_x(r,j,ux,uz).b_y(r,j)+\partial\sum_j F_z(r,j,ux,uz).b_x(r,j)}{\partial\{\delta\}_r}.$$

$$\frac{\partial M_z}{\{\delta\}_r}=\frac{\partial\sum_j F_x(r,j,ux,uz).b_z(j)+\partial\sum_j F_y(r,j,ux,uz).b_x(r,j)}{\partial\{\delta\}_r}.$$

where:

$$F_z(r,j,ux,uz)=\sum_r K_i.\delta_i(r,j,ux,uz).cos(\alpha_i(r,j,ux,uz))cos(\psi(r,j)))$$

$$F_y(r,j,ux,uz)=\sum_r K_i.\delta_i(r,j,ux,uz).cos(\alpha_i(r,j,ux,uz))sin(\psi(r,j)))$$

$$F_x(r,j,ux,uz)=\sum_r K_i.\delta_i(r,j,ux,uz).sin(\alpha_i(r,j,ux,uz))$$

5. The *(r)* pair bearings life

Using the Lundberg-Palmgreen life rating method applied to the (*r*,*j*) pair bearings, the basic dynamic element capacity, Q_c, is defined as the ball load which will result in a life of a million revolutions of the raceway with 90 percent probability of survival.

For a ball with diameter 25mm, basic dynamic ball capacity can be calculated as:

$$Q_c = A.\lambda.\left(\frac{2.f}{2.f-1}\right)^{0.41}\frac{(1\mp\gamma)^{1.39}}{(1\pm\gamma)^{1.3}}\left(\frac{Dw}{dm}\right)^{0.3}.D_w^{1.8}.N_r^{-1/3} \quad (27)$$

where: $A=98;\ \lambda=1;$

The calculation of the inner and outer race lives are given in Eq. (28), for the case of the inner race rotating with respect to the load. The race lives are calculated per Eq. (29), for the case when the inner race is stationary with respect to the load. The combination of the race lives to give the bearing life is shown by Eqs. (30) and (31). If the inner race rotates with respect to load, the lives can be calculated as:

$$L_r(r)=\left(\frac{Q_{cr}}{Q_{er}}\right)^4 \qquad L_s(r)=\left(\frac{Q_{cs}}{Q_{es}}\right)^4 \quad (28)$$

where:

$$Q_{er}(r)=\left[\frac{\sum_{j=0}^{Nr-1}Q(r,j)^p}{Z}\right]^{\frac{1}{p}};\ Q_{es}(r)=\left[\frac{\sum_{j=0}^{Nr-1}Q(r,j)^{pe}}{Z}\right]^{\frac{1}{pe}} \quad (29)$$

The *(r)* bearing life in terms of millions of revolutions can be calculated as:

$$L_{10}(r)=(L_r^{-e}(r)+L_s^{-e}(r))^{-1/e} \quad (30)$$

Results that the pair bearings life in million of revolutions is:

$$L_{10}=(L_{10}^{-e}(1)+L_{10}^{-e}(2))^{-1/e} \quad (31)$$

Bearing life in term of hour can be calculated as:

$$B_{10}=\frac{L_{10}.10^6}{60.RPM} \quad (32)$$

6. Numerical examples

The following geometrical values have been considered:

$B1=B_{i1}=B_{i2}=10$ [mm]; $B2=B_{o1}=B_{o2}=10$ [mm]
$L2=L1=10$ [mm]; $N_r=12$ balls / bearing;
$Dw=9.525$ [mm]; $dm=46$ [mm]; $\alpha_o=15$ [deg]
$Ro/Dw=0.52$; $Ri/Dw=0.53$; $L=50$ [mm]

The effect of the "*ja*" parameter versus *Fx1* and *Fx2* evolution is related in Fig. 6. The external forces are: *Fz=10* [N], *Fy=10* [N], *Fx=200* [N], *L=50* [mm]

To point out the influence of the length tolerances on load distribution, the axial displacement is related in Fig. 7 as function of the "*ja*" parameter.

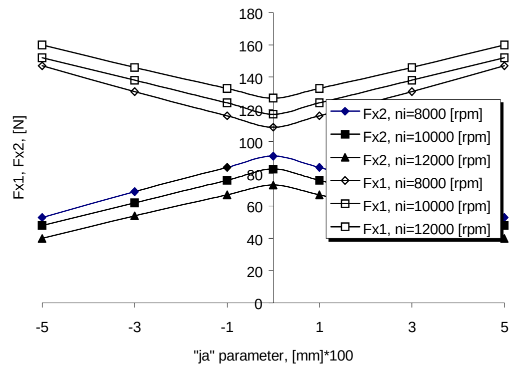

Fig. 6. "*ja*" parameter versus axial displacement

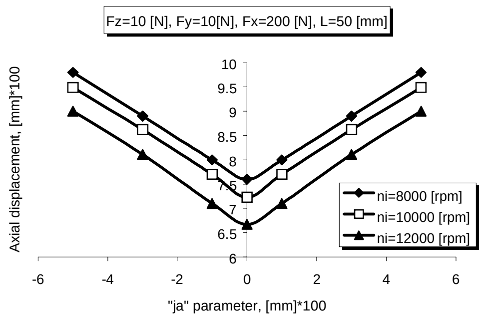

Fig. 7. Axial displacement versus "*ja*" parameter for the "*r*" bearing

The influence of the length tolerances on the bearing life is related in Fig. 8 and 9, as function of the "*ja*" parameter. The external conditions are: *Fz=5*[N], *Fy=0* [N], *Fx=500* [N]; *RPM=25000* [rpm]., *L=100* [mm].

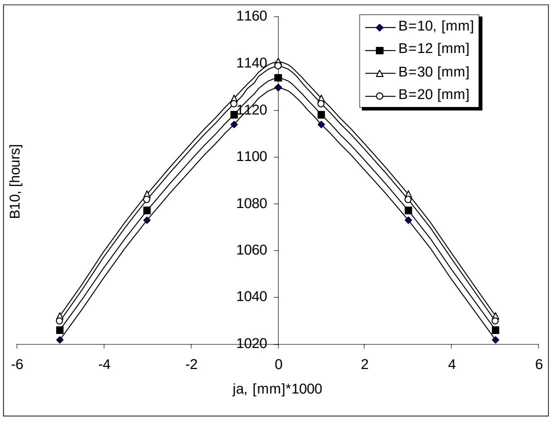

Fig.8. Pair bearing's life versus "ja" parameter

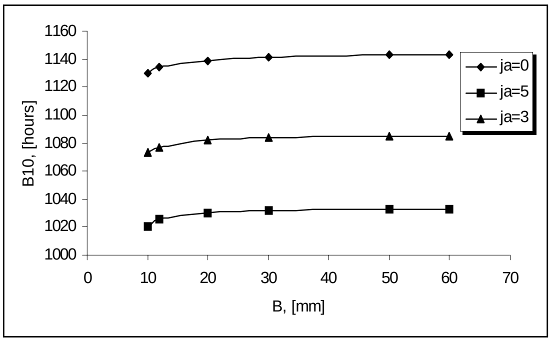

Fig.9. Pair bearing's life versus "*B*" parameter

6. Conclusions

The parameters "*ja*" and "*B*" influnce the components of the load vector *{F}*$_r$, and also the load distribution in the "*r*" rolling bearings' arrangement.

- From the viewpoint of the maximum rating life, an optimum distance between the curvature centres of the races of the two bearings has been found (Fig.8 and 9).
- In order to obtain a maximum rating life for a tandem mounted bearings (Fig.1), the parameter "*ja*" must approach to zero.

Notations

Indexes, distances and coordinate systems

r	bearing index
j	ball number in the "r" bearing
i	inner ring
o	outer ring
n	point contact constant, n=1.5
N_r	the number of balls for the "r" bearing
m	ball weight ,[kg]
K_{ech}, $K_{i,}$, K_e	equivalent, inner, and outer, rigidity factors
dm	pitch diameter, [mm]
Dw	ball diameter, [mm]
O_w	mass center of the (r,j) ball;
$O_{i,e}$	centers of curvature
L1,L2	lengths of intermediate parts
$TL_{1,2}$	tolerances of the L1 and L2 lengths, [mm]
B1, B2	"r" bearing width, [mm]
$B_{i,r}$, $B_{o,r}$	inner and outer ring width, [mm]
R, B, C, L	lengths, [mm]
D1, D2	initial distances between curvature centers
D_{1p}, D_{2p}	final distances between curvature centers
$R_{o,i}$	outer and inner raceway radius, [mm]
$TB_{i,o,r}$	length tolerances of the inner and outer rings,
OXYZ	inertial system
$OX_1Y_1Z_1$	rolling element frame for the (r,j) ball
ux,uz	displacement of the mass center
x,y,z	index to describe the axes
l_{oe}	distance between O_e and O_w points, [mm]
l_{oi}	distance between O_i and O_w points, [mm]
L_{ie}, $L_{ie}(r)$	distance between O_i and O_e points, [mm]
Sd, Sd1(r)	diametrical clearance of the bearing [mm]
Q(r,j)	normal load, [N]
$Q_{i,o}(r,j)$	normal load –inner and outher racevay
Fmi, Fmo, Ffi, Ffo	tangential forces
ja	axial clearance, [mm]
{}	vector index

Forces and moments

{F}, $\{F\}_r$	force vectors
{M}	moment vector
Mg	gyroscopic moment
F_x, F_{xr}	axial load along OX axes;
F_y, F_{yr}	radial loads along OY axes
F_z, F_{zr}	radial load along OZ axes
M_y, M_z, M_{yr}, M_{zr}	external moments which act around of the OY and OZ axis respectively

Life parametrs

L_{10}	bearing life, mr, 90% prob. survival
B_{10}	bearing life, hours, 90% prob. survival
Q_c	dynamic capacity
Q_{er}	dynamic equivalent load of the rotating race
Q_{es}	dynamic equivalent load of the stationary race
RPM	revolutions per minute

Greek notations

$\alpha_{0,}$	initial contact angle
$\alpha_{0,r}$, α_{0p}	initial contact angle affected by "ja" parameter
$\alpha_{r,1}$	initial angle between "R" and OZ axis
$\alpha_1(r,j)$	final angle between "R" and OZ axis
$\alpha_s(r,j)$	contact angle obtained from static equilibrium
$\alpha_i(r,j)$	inner contact angle obtained from the static equilibrium
$\alpha_e(r,j)$	outer contact angle obtained from the static equilibrium
$\alpha_i(r,j,ux,uz)$	outer contact angle
$\alpha_e(r,j,ux,uz)$	inner contact angle
$\delta_i(r,j,ux,uz)$	local contact deformation at the inner ring level for the (r,j) index
$\delta_i(r,j,ux,uz)$	local contact deformation at the outer ring level for the (r,j) index
γ	γ=Dw/dm, dimensionless parameter
μ	friction coefficient
λ	John's coefficient
$\omega_{c,w}$	angular speed of the cage and ball,

REFERENCES

1.**T.Harris,** „Rolling Bearing Analysis”. *John Wiley & Sons*, New York, London, Sydney, 1966

2**.M.D.Gafitanu, D.N. Olaru, M.C. Cocea**, „Die Veluste wegen der Reibung in Radial-Axial-Kugellagern bei hohen Drehzahlen“, *Weqr,* 160 (1993), 51-60

3.**M.D. Gafitanu, Cretu, Sp, Olaru D** – „Rulmenti, Proiectare si tehnologie“, *vol 1*, Ed. tehnica, Bucuresti, 1985

4.**P.K. Gupta**, „Dynamics of Rolling-Element Bearings, Part III: Ball Bearing Analysis”, *Transactions of the ASME, vol. 101,* July 1979

5.Rumbarger, J.H. Poplawski, J., V, „Correlating Computerized Rolling Bearing Analysis” *Techniques to the ISO Standards on Load Rating Life, Tribology Transactions, vol.37* (1994), 793