SOLUTION TO DESCRIBE THE INTERNAL KINEMATICS IN BALL BEARINGS WITH 2, 3 OR 4 CONTACT POINTS

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ABSTRACT.

The 3 or 4 contact point ball bearings, have the special inner and / or outer rings geometry. Such bearings may operate smoothly from 2 to 3 or 4 contact points while changing operating conditions. For ball bearings with up to 2 point contacts, the control criteria of ball bearing under the inner or outer raceway is unusable. The paper presents a mathematical model to describe the ball internal kinematics under the effect of the external working conditions. A computer code named SRB-4PCBB was developed to 2, 3 and 4 point contact ball bearings analysis. The model analyses the internal kinematics of the ball by the principle of power minimization tackeing into account the he gyroscopic and spin movement effect in the ball equilibrium. Because in this paper is presented only the quasi-static equilibrium, the cage and ball angular speed are taken as constants values

KEYWORDS: arched race ball bearings, internal kinematics, 2 to 4 point contact ball bearings, spin and gyroscopic movement, ball equilibrium.

1. INTRODUCTION

In the ball bearings with arched-outer-race the centrifugal forces is shared between the two ball-outer race contacts above some transition speed. Therefore, when the arched bearing has two contact points per ball at the outer race, the centrifugal loading can be shared and thereby reduce the maximum pressure at the outer ring contacts that subsequently will increase the bearing life. A first analysis of an arched bearing design was performed by Hamrock and Anderson [1] who indicated the possibility of significant fatigue life improvement. They developed an quasi-static analysis of an arched-outer race ball bearing considering only ball centrifugal forces, neglecting gyroscopic. The model was later improved [2] by adding the effect of the gyroscopic moment acting on the balls.

Recent investigations on high-speed lubricated ball bearings have revealed that the power loss by ball spinning is shared between the inner and the outer race [3]. The present paper proposes a mathematical model to describe the ball equilibrium of ball bearings with 2, 3 or 4 contact points. The bearing element kinematics has been solved considering a mixed control of the ball between the bearing raceways. The main mathematical considerations concerning the computing method are presented in [3].

2. GEOMETRY CONSIDERATIONS

The model has been developed imposing the outer ring(s) fixed in space. Figure 1 presents a simplified cross-section of a four points contact ball bearing (4PCBB), pointing out the position of the raceway curvature centers (points P_{idx}) relative to the ball mass center (point Ow). The co-ordinate system, external load vector *{E}* and deformation vector *{δ}* are presented in figure 2. The definition of each symbol is given in the nomenclature.

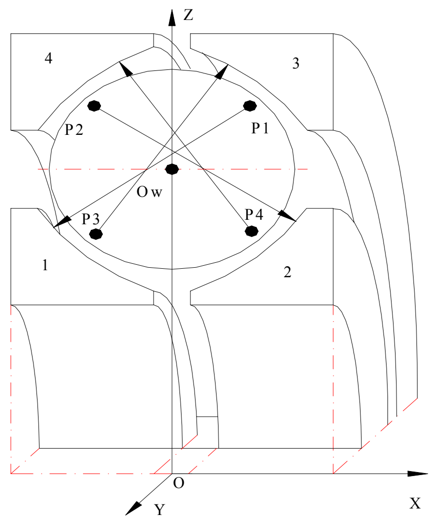


Fig. 1. Schematic view of a 4 points contact ball bearing (4PCBB).

Under external applied loads or imposed rotations and translations of the inner ring the raceway curvature centers reach their final positions noted $P_{idx}f$, as schematically presented in figure 3. For each ball numbered by j=1 to Z, the nominal and operating contact angles, respectively α_{idx} and β_{idx}, have their origin coincident with the OZ-axis.

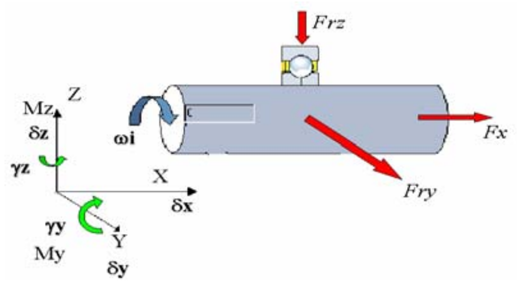


Fig. 2 – Co-ordinate system, external applied loads and corresponding rotations and translations.

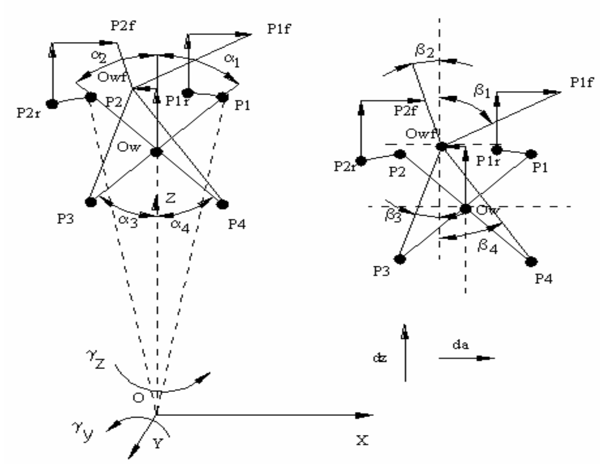


Fig. 3 – Initial and final positions of curvature centers.

Table 1. Matrix code for all ball bearings type.

4PCBB-1234	idx	sdux	sduz	sdx	sdz
	1	1	1	1	1
	2	-1	1	-1	1
	3	-1	-1	0	0
	4	1	-1	0	0
4PCBB-13	idx	sdux	sduz	sdx	sdz
	1	1	1	1	1
	2	0	0	0	0
	3	-1	-1	0	0
	4	0	0	0	0
4PCBB-123	idx	sdux	sduz	sdx	sdz
	1	1	1	1	1
	2	-1	1	-1	1
	3	-1	-1	0	0
	4	0	0	0	0
4PCBB-134	idx	sdux	sduz	sdx	sdz
	1	1	1	1	1
	2	0	0	0	0
	3	-1	-1	0	0
	4	1	-1	0	0

The unitary mathematical analysis of different types of ball bearing with 2 to 4 point contacts is reached by using a matrix code. In this procedure all ball bearing types are derived from the more general geometry of a 4 point contact ball bearing, call type 4PCBB-1234. The matrix code is given in table 1, where idx denotes the contact points [4, 6].

QUASI-STATIC ANALYSIS CONSIDERATIONS

Under the effect of the external load the inner ring(s) of the 4PCBB structure is (are) displaced. This displacement produce at the ball-raceways interfaces a load distribution. The normal load and the elastic deflection of each point contact between ball "j" and raceway "idx" are given according with Hertz theory [5].

$$Q_{j,idx} = K_{idx}.\delta_{j,idx}^{1.5} \qquad (0)$$

where:

K_{idx} represents the contact stiffness.

δ_{idx} represents the elastic deflection in the ball / ring contact, $\delta_{j,idx} \leftarrow \delta_{j,idx}(idx, ux_j, uz_j, \delta x, \delta z, \delta y, \gamma z, \gamma y)$

Ball equilibrium

Using the significance term for sdx, sdz, sdux, sduz presented in Table 1 and **[3,5]**, two equilibrium equations are written for each ball according to Eq.1. The sign effect of the gyroscopic movement is introduced by using the smgz, smgx coefficients presented in **[3, 5]**

$$\sum_{idx} Q_{j,idx}\cdot\left[\begin{array}{c} \sin(\beta_{j,idx}).sdux_{idx} \\ -\cos(\beta_{j,idx}).smgx_{idx}.CFB_j(\beta w)\end{array}\right]=0$$

$$\sum_{idx} Q_{j,idx}\cdot\left[\begin{array}{c} \cos(\beta_{j,idx}).sduz_{idx} \\ +\sin(\beta_{j,idx}).smgz_{idx}.CFB_j(\beta w)\end{array}\right]+Fc=0 \qquad (1)$$

with:

$Q_{j,idx} = Q_{j,idx}(ux_j,uz_j)$ = contact load

$\beta_{j,idx} = \beta_{j,idx}(ux,uz)$ = contact angles, according to [4].

βw – the ball angle

The Newton-Raphson iterative method is applied to find the new components of the ball mass center displacements $(ux, uz)_j$ as function of the components of the previous vector {δ}.

The $CFB_j(\beta w)$ functions are:

$$CFB_j(\beta_w) = \frac{2}{dw}\frac{M_{gyr}(\beta w)}{\sum_{idx} Q_{j,idx}} \qquad (2)$$

where:

$$M_{gyr}(\beta w) = \rho_{bille}.Dw^5.\omega c.\omega b_j.|\sin(\beta w)|.\frac{\pi}{60} \qquad (3)$$

ωc = cage angular speed

ωb_j = ball angular speed

Dw= ball diameter

ρ_{ball}= specific weight of the ball

The $CFB_j(\beta_w)$ coefficient results by following the next algorithm:

$\sum_{idx} \lambda_{idx} = \lambda = 1$, and

$$\lambda.M_{gyr}(\beta w) = M_{gyr}(\beta w) = CFB_j(\beta w).\frac{dw}{2}.\sum_j Q_{j,idx}$$

$$\lambda_{idx}.M_{gyr}(\beta w) = CFB_j(\beta w).\frac{dw}{2} Q_{j,idx}$$

By summation

$$M_{gyr}(\beta w).\sum_{idx} \lambda_{idx}. = CFB_j(\beta w).\frac{dw}{2}.\sum_{idx}.Q_{j,idx}$$

$$M_{gyr}(\beta w) = CFB_j(\beta w).\frac{dw}{2}.\sum_{idx}.Q_{j,idx}\ , \rightarrow \text{Eq. 2}$$

Pseudo code

initial solution
CFB ← 0; βw ← 0
Solve Eq.1 : ECFA=0; ECFR=0; => Q_{idx}, β_{idx}
REPEAT
    $Q0_{idx}$ ← Q_{idx}
    Compute Eq.2 => CFB(βw,Q_{idx})
    Solve Eq.1 => Q_{idx}(CFB)
UNTIL $\left|Q_{idx} - Q0_{idx}\right| < eps$

INTERNAL KINEMATICS

Assumption of mixed control of the ball

Figure 4 shows the necesity to develop a new mathematical model because the inner or outer raceway control can not be assumed for all ball bearings type.

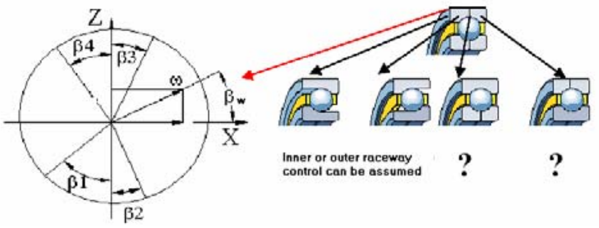


Fig.4. Mixed ball bearing control necesity

Sliding speeds

The speed vector at point P of the contact ellipse V_P has three components as shown in Fig. 6 :

$$\vec{V}_P = \vec{V}_{sP} + \vec{V}_{yP} + \vec{V}_{xP} \quad (4)$$

where V_{sP} is the linear sliding velocity due to the spin movement ($V_{sP} = \omega_s \times r$) with ω_s the spin angular rotational speed and r the distance from the ellipse center; V_{yP} is the linear speed in the rolling direction; V_{xP} is the linear speed in the transverse direction.

The present model uses this hypothesis and the criteria of minimum power losses around the ball to deduce the direction of the ball angular rotational speed vector (defined by angle β_w in figure 5) [4, 6].

The hypothesis of minimum power dissipated by ball "j" is written as [3, 5]:

$$\frac{\partial Pf}{\partial \beta_w} = \frac{\partial Px}{\partial \beta_w} + \frac{\partial Py}{\partial \beta_w} = 0 \quad (5)$$

where:
P_x represents the power loss due to sliding along the rolling direction;
P_y represents the power loss due to sliding along the transverse (axial) direction.

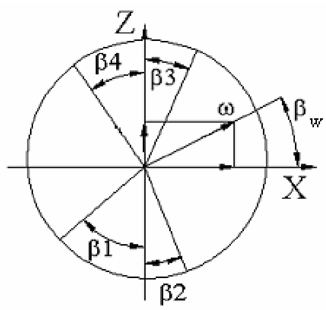


Fig. 5 – Contact angles β_1, β_2, β_3, β_4 and ball axis attitude angle β_w.

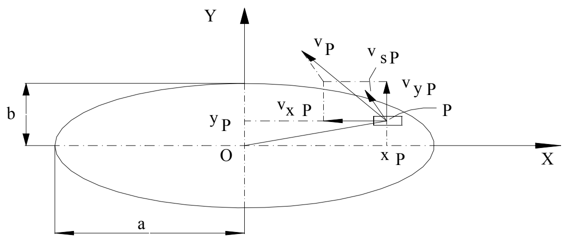


Fig. 6 – Linear speed components for point P.

Assuming that the friction coefficient is uniform over the contact area eq. (5) becomes:

$$\frac{\partial P}{\partial \beta_w} = \frac{\sum_{idx=1}^{4} T_{idx} \cdot \sum_{\xi=-a}^{a}\left[B_{\xi,idx}\cdot\left(A_{idx,\xi}.\omega s_{idx} - \xi.\omega r_{idx}.sduz_{idx}\right).q_{idx,\xi}\right]}{\sum_{idx=1}^{4} Q_{idx}.b_{idx}.T_{idx}.\omega r_{idx}\cdot\left(\frac{\left|sin(-u_{idx}+\beta_w)\right|}{sin(-u_{idx}+\beta_w)}\right)} + \quad (6)$$

where:

$$A_{idx,\xi} = \left[\sqrt{Rech_{idx}^2 - a_{idx}^2} + \sqrt{\left(\frac{Dw}{2}\right)^2 - \xi^2} - \sqrt{\left(\frac{Dw}{2}\right)^2 - a_{idx}^2}\right]$$

$Rech_{idx} = 2\frac{r_{idx} \cdot R}{r_{idx} + R}$ R = Dw/2

$r_{idx} = Dw\ .\ f_{idx}$ $\xi_{idx} \subset [-a_{idx}, +a_{idx}]$

$$B_{\xi,idx} = \frac{\left|Vrg_{\xi,idx} - Vw_{\xi,idx}\right|}{Vrg_{\xi,idx} - Vw_{\xi,idx}}$$

$u_{idx} = \{-\beta_1, \beta_2, -\beta_3, \beta_4\}$

$\omega r_{idx} = \omega.cos\left(u_{idx} - \beta_w\right)$ the rolling component of the ball angular rotational speed at contact "*idx*";

$\omega s_{idx} = \omega.sin\left(\beta_w - u_{idx}\right)$ the spin component of the ball angular rotational speed at contact "idx";

Vw is the linear speed of point P on ball "*j*", function of parameters (*j, idx, ξ*):

$$\left\{Vw_{\xi,idx}\right\} = A_{idx,\xi}.\omega r_{idx} + \xi.\omega s_{idx}.sduz_{idx}$$

Vrg is the linear speed of point ξ on raceway "idx":

$\omega_{idx} = \{\omega_i, \omega_i, \omega_o, \omega_o\}$.

$$\left\{Vrg_{\xi,idx}\right\} = \left[\frac{dm}{2} - sduz_{idx}\left(A_{idx,\xi}cos(\beta_{j,idx}) - \xi\, sin(\beta_{j,idx})sdux_{idx}\right)\right] * \left(\omega_{idx} - \omega_c.sduz_{idx}\right)$$

The sliding velocity for point *P(ξ)* of contact "*idx*", is the difference of the absolute velocities:

$$\{Vsli\}_{P,idx}=[\{Vrg_{P,idx}\}-\{Vw_{P,idx}\}].T_{idx}.sduz_{idx} \qquad (7)$$

where: $T_{idx}=0$ if $Q_{idx}=0$; or $T_{idx}=1$ if $Q_{idx}>0$;

If the contact load respects the Hertz theory, Q_{idx} and $q_{idx,\xi}$ are given as:

$$Q_{idx} = \sum_{\xi=-a}^{a} \sigma_{\xi} \cdot \frac{\pi \cdot a_{idx} \cdot b_{idx}}{NSa \times 1.17794} = \sum_{\xi=-a}^{a} q_{idx,\xi}$$

$$\sigma_{\xi} = \frac{Q_{idx} \cdot 1.5}{\pi \cdot a_{idx} \cdot b_{idx}} \cdot \sqrt{1-\left(\frac{\xi}{a_{idx}}\right)^2}$$

If non – Hertzian contact occur, Q_{idx} and $q_{idx,\xi}$ are given according with [3, 6], as:

$$q_{idx,\xi} = E0.k^{-0.11} \cdot \delta_{\xi} \cdot \Delta_{\xi} \cdot f \cdot Q(k_{idx,\xi})$$

$$fQ(k_{idx,\xi}) = \frac{0.94896 - 0.09445.\ln(k_{idx,\xi})}{1 + 0.45412.\ln(k_{idx,\xi})}$$

Finally eq. (6) gives the ball axis attitude angle β_w.

APPLICATIONS

The mathematical model is applied to 4PCBB-1234, 4PCBB-134, 4PCBB-123 and 4PCBB-13 bearing types by imposing constants values for cage and ball speeds, and also for the main ball bearings parameters. The modified parameters are the axial and radial displacement of the inner ring(s). The input data are presented in table 2. Four analyses (A1, A2, A3, A4) are presented in what follows.

For all tests the angular speeds for the cage (ωc) and for the idx balls are given as: $\omega c=\omega i.(1-\gamma)/2$ and $\omega b=\omega i.(1-\gamma^2).Dm/Dw/2$, with $\omega i=ni.\pi/30$ and $\gamma=Dw/dm$. The results are presented in table 3, as follows

Table 2. Constants. Internal geometry

Application example	**A1**	**A2**	**A3**	**A4**
Bearing type	4PCBB-1234 	4PCBB-134 	4PCBB-123 	4PCBB-13 
Ball diameter, Dw (mm)	20			
Bearing pitch diameter, dm (mm)	150			
Inner ring curvature factor, fi	0.525			
Outer ring curvature factor, fe	0.51			
Inner ring shim angle (deg)	20	0	20	20 (contact angle)
Outer ring shim angle (deg)	30	30	0	20 (contact angle)
Angular speeds, rpm	ni=1000			ni=20000
Radial displacement of the IR, mm	dz=0,08			
Axial displacement of the IR, mm	dx=0.05; dx=0.03 and dx=0			

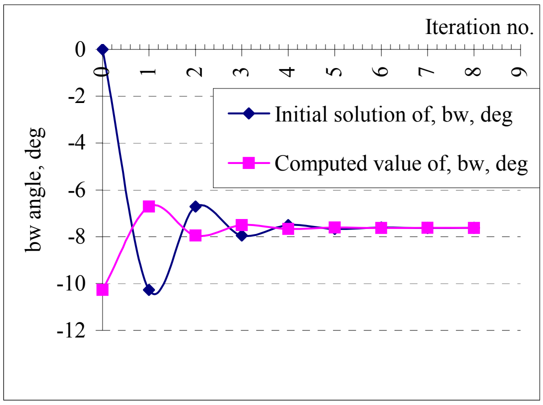
Table 3. Results for A1, A2, A3 and A4 cases.

Test type		Contact load, N				Contact angle, deg				Inclination angle of the ball angular rotational speed, deg
TEST A1	dx	idx=1	idx=2	idx=3	idx=4	idx=1	idx=2	idx=3	idx=4	βw
	-0.05	6359	14516	7863	14046	14.85	21.9	21.99	28.62	10
	-0.03	7672	12556	8777	12470	16.3	20.53	23.4	27.4	5.97
	0	9932	9932	10437	10437	18.44	18.44	25.44	25.44	0
TEST A2	dx	idx=1	idx=2	idx=3	idx=4	idx=1	idx=2	idx=3	idx=4	βw
	-0.05	14390	0	9245	9735	5.87	0	27.56	24.6	-5.8
	-0.03	13864	0	8446	6990	3.5	0	27.03	25.29	-3.1
	0	13577	0	7568	7568	0	0	26.2	26.2	0
TEST A3	dx	idx=1	idx=2	idx=3	idx=4	idx=1	idx=2	idx=3	idx=4	βw
	-0.05	10686	6258	16128	0	20.616	16.55	7.05	0	-6.1
	-0.03	9547	6930	15644	0	19.81	17.4	4.26	0	-3.7
	0	8107	8107	15371	0	18.62	18.62	0	0	0
TEST A4	dx	idx=1	idx=2	idx=3	idx=4	idx=1	idx=2	idx=3	idx=4	βw
	-0.05	7038	0	9013	0	15.4	0	11.9	0	-13.22
	-0.03	8056	0	10017	0	16.86	0	13.49	0	-14.19
	0	9845	0	11785	0	18.95	0	15.74	0	-17.202

For a 4PCBB-1234 bearing,, the algorithm convergence was tested according to the "Pseudo code" for 2 different cases assuming a contact radial load:

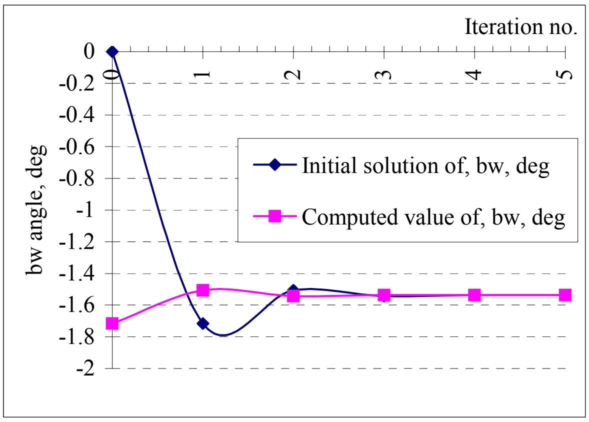
- for a High axial load

Iter no.	Initial sol., βw, deg	Q1	Q2	Q3	Q4	Computed value of, βw, deg
0	0	413.3	0	1422.4	990.3	-10.27
1	-10.27	406.1	0	1275.3	1143.4	-6.71
2	-6.71	408.1	0	1324.1	1088.7	-7.95
3	-7.95	406.6	0	1305.9	1107.8	-7.50
4	-7.50	407.3	0	1312.6	1100.9	-7.67
5	-7.67	406.9	0	1310.0	1103.4	-7.60



- for Low axial load

Iter no.	Initial solution bw, deg	Q1 [N]	Q2 [N]	Q3 [N]	Q4 [N]	Comp. value of, bw, deg
0	0	403.82	314.8	1405.9	1318.5	-1.71
1	-1.71	407.89	309.5	1389.9	1333.5	-1.50
2	-1.50	407.68	310.0	1392.1	1331.5	-1.54
3	-1.54	407.69	309.9	1391.7	1331.9	-1.53
4	-1.53	407.69	309.9	1391.8	1331.8	-1.53



CONCLUSIONS

The numerical results confirm the mixed control criteria presented by Nelias [3]. To evaluate the complex kinematics in the 2, 3 or 4 point contact ball bearing a computer code was developed. It implements a unitary mathematical model with 5DOF able to describe the quasi-static and quasi-dynamic behavior of lubricated ball bearings [6].

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