# APPROXIMATE SOLUTIONS TO HERTZIAN AND NON-HERTZIAN CONTACT ELASTICITY 

Daniel REZMIREŞ ${ }^{1}$, Cezar RACOCEA ${ }^{2}$, Daniel NELIAS ${ }^{3}$<br>${ }^{1}$ S.C. FORPRES S.A. Iasi, ${ }^{2}$ Technical University "Gh. Asachi" of Iasi, ${ }^{3}$ INSA - Lyon drezmir@yahoo.com


#### Abstract

This paper presents a new numerical method to calculate the Hertzian and NonHertzian contact. Results calculated with the new relations are successfully compared with data from literature. The proposed method assures a good continuity of the transition between point and modified point contact.


KEYWORDS: Hertz contact, non-Hertz contact, cutting point contact analysis, Borland Delphi, Compaq Visual Fortran.

## 1. INTRODUCTION

To study the Quasi- Static parameters for a NonHertzian contact the classical methods can be uses. In this case, a computing system must allocate to that duty a large memory resources. That implies also a big consuming time. To reduce these disadvantages some interpolation functions was create. A computing code was developed in Borland Delphi and Visual Fortran for a French company and some results are shown.

## 2. NUMERICAL FORMULATION.

When a bearing is loaded some conjunctions can be of line contact type and others of point contact type [1]. The contact load is a function of the center of mass displacement of the rolling element $(\xi)$. The local contact deformation for a slice " j " is given as the geometrical interference between the roller element and raceway geometry as:

$$
\begin{equation*}
\delta_{j}=\left(\frac{1}{R w}-\frac{1}{R c}\right) \cdot \frac{X R_{j}^{2}}{2}+\xi \tag{1}
\end{equation*}
$$

where:

$$
X R_{j}=\frac{2 \cdot j-N}{N} \cdot \frac{l w}{2}
$$

$j=$ is the slice index,
$R w=$ local rolling element radius profile,
$R c=$ local raceway radius.
The proposed functions to obtain the contact parameters are given by equation (2), as follows [4]:
Local contact pressure, $P=P(j)$

$$
\begin{equation*}
P_{j} \approx \frac{0.282 \cdot E \cdot k^{-0.11} \cdot \delta_{j} \cdot 2}{\pi \cdot b_{j}} \cdot f p(k) \tag{2a}
\end{equation*}
$$

Local semi-width, $b=b(j)$

$$
\begin{equation*}
b_{j}=R_{y_{j}} \cdot \sqrt{\frac{\delta_{j} \cdot k^{-0.11}}{R y}} \cdot 1.15617 . f b(k) \tag{2b}
\end{equation*}
$$

Local load, $Q=Q(j)$

$$
\begin{equation*}
Q_{j}=E 0 . k^{-0.11} . \delta_{j} \cdot \Delta x_{j} \cdot f Q(k) \tag{2c}
\end{equation*}
$$

with:

$$
\begin{aligned}
& f p(k)=\frac{3.2821-0.3322 \cdot \ln (k)}{1+0.42877 \cdot \ln (k)} \\
& f b(k)=\frac{1.21386-0.07678 \cdot \ln (k)}{1+0.115078 \cdot \ln (k)} \\
& f Q(k)=\frac{0.94896-0.09445 \cdot \ln (k)}{1+0.45412 \cdot \ln (k)}
\end{aligned}
$$

and

- $\Delta x_{j}=\frac{l w}{N}$, length of the slice section " $j$ "
- $l w=$ the rolling element length,
- $k$, the contact elipticity [1,3],
- $E_{o}$, the equivalent modulus of elasticity of the two bodies in contact [1,3]


## 3. NUMERICAL APPLICATIONS

The model was applied to study the Hertz and nonHertz contact type .

### 3.1. Hertz contact type

Assuming a spherical roller bearing (SRB) 22308C with

- Contact angle $14.33^{\circ}$;
- Pitch diameter $d m=66[\mathrm{~mm}] ;$
- Roller diameter $d w=13[\mathrm{~mm}]$;
- Roller length $l w=12[\mathrm{~mm}]$;
- Roller radius $R w=39.5$ [mm]
- Inner raceway profile radius $R c=40.35$ [mm]

Table 1 shows some numerical comparisons between the numerical solutions gives by Hertz theory and the proposed formulas (see also figure 1).

Table 1. Numerical comparisons, SRB 22308 C - roller - inner ring contact.

| Contact <br> load | Contact <br> ellipticity | Hertz theory |  | Eq. 2 |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | k(Rw) | Maximum <br> pressure [MPa] | $\mathrm{b}[\mathrm{mm}]$, <br> semi width of the <br> point contact | $\max (\mathrm{P}),[\mathrm{MPa}]$ | $\max (\mathrm{b}),[\mathrm{mm}]$ |
|  |  | 730.9 | 0.06653 | 731.59 | 0.06652 |
| 300 | $\mathrm{k}=44,27565$ | 1481.8 | 0.13488 | 1483.2 | 0.13487 |
| 2500 | $\mathrm{Rw}=39.5$ | 1029.3 | 0.0932 | 1029.1 | 0.0932 |
| 300 | $\mathrm{k}=16.0082$ |  |  |  |  |
| 2500 | $\mathrm{Rw}=36.5$ | 2086.9 | 0.1980 | 2086.5 | 0.1980 |



Fig. 1a. The pressure distribution according to the table 1, and geometry input data.


Fig. 1b. Pressure distribution - details. Numeric results for Rw=39.5.

### 3.2. Non Hertz contact type. Example for the multi radius profile

For the case of the multi-radius profile was chosen the case given in the [2] \& [3] references. In figure 2, the

bearing geometry and the pressure distribution are presented for two distinct cases as a function of the external load. The numerical validation of the proposed mathematical model is assured by the results presented in table 2 and figures $3 \mathrm{a}, 3 \mathrm{~b}, 3 \mathrm{c}$ respectively.

Fig. 2. Ref [2] \& [3]. Rroller diameter, $\mathrm{Dw}=15 \mathrm{~mm}$, roller length, $\mathrm{lw}=16 \mathrm{~mm}$, the race diameter, $\mathrm{d}=58.5$.
Table 2. Numerical results. Comparisons with reference [2].

| Load, N | Krzeminski [2] | Half space model | FEM | Full model | Eq. (2) |
| :--- | :--- | :--- | :--- | :--- | :---: |
| 10000 | 002785 | 0.028 | 0.02444 | 0.02482 | $\mathbf{0 . 0 2 2 5 6}$ |
| 33800 | 0.06714 | 0.0675 | 0.0570 | 0.05737 | $\mathbf{0 . 0 6 6 5 3}$ |



Fig. 3a. The pressure distribution imposing the contact load as 33800 N .


Fig. 3b. Pressure distribution - details. Numerical results.


Fig. 3c. Contact demi-width

The pressure distribution presented in figure 2a, and the center mass displacement presented in table 2, give a good correlation between author's relations and references [2] and [3], respectively.

## 4. CONCLUSIONS

The proposed equations have been compared successfully to the different method [2] and [3], giving confidence in the new suggested method. The proposed method improves the computing speed of the nonHertzian cutting point contact parameters and shows that the imposed hypothesis of the linear dependence between load and centre of mass displacement of the rolling element gives good results.

## REFERENCES

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